Allocation of Transportation Resources

Presented by:
Anteneh Yohannes

Problem

- State DOTs must allocate a budget to given projects
- Budget is often limited
- Social Welfare Benefits
- Different Viewpoints (Two Players)
  - Planners
  - Road Users

Purpose of Project

- Produce a method of prioritizing projects
  - Consider Total System Travel Time (TSTT) performance measure
- Allocate the budget to priority links
- Provide State DOTs with a decision making tool

Methodology

- Bi-Level Optimization

- Planners
  - Upper Level Problem (ULP)
  - Minimize Total System Travel Time (TSTT)

- Users
  - Lower Level Problem (LLP)
  - Traffic Assignment (UE)

Data Required

- Number of Links in the network
  - Capacity
  - Length
  - Free Flow Travel Time
  - Alpha and Beta parameters
  - Connecting Nodes
- O/D Matrix
- Budget

Formulation

Upper Level problem (ULP)

Objective Function :

$$\textbf{Minimize TSTT} = \sum_{a} x_a \, t_a \, (x_a, y_a)$$

Subject to:

$$\sum_{\forall_a} g_a(y_a) \leq B$$

$$y_a \geq 0 : \forall \, a \in A$$

Where:

$TSTT$	:	Total System Travel Time
x_a	:	Flow for link a
y_a	:	Capacity expansion for link ‘a’ (nonnegative real value)
t_a	:	Travel time for link a
$t_a(x_a, y_a)$	:	Travel cost on link a as a function of flow and capacity expansion
$g_a(y_a)$	:	improvement cost function for link ‘a’
B	:	Budget (nonnegative real value)

Formulation

Lower Level problem (LLP)

$$\text{Minimize TT} = \sum_{a \in A} \int_0^{x_a} t_a(x_a, y_a)dx$$

Subject to:

$$q_{ij} = \sum_{k \in k^{ij}} f_k^{ij} \; \forall (i,j) \in IJ$$

$$x_a = \sum_{(i,j) \in IJ} \sum_{k \in K^{ij}} \delta_{ak}^{ij} f_k^{ij}, \quad \forall_a \in A$$

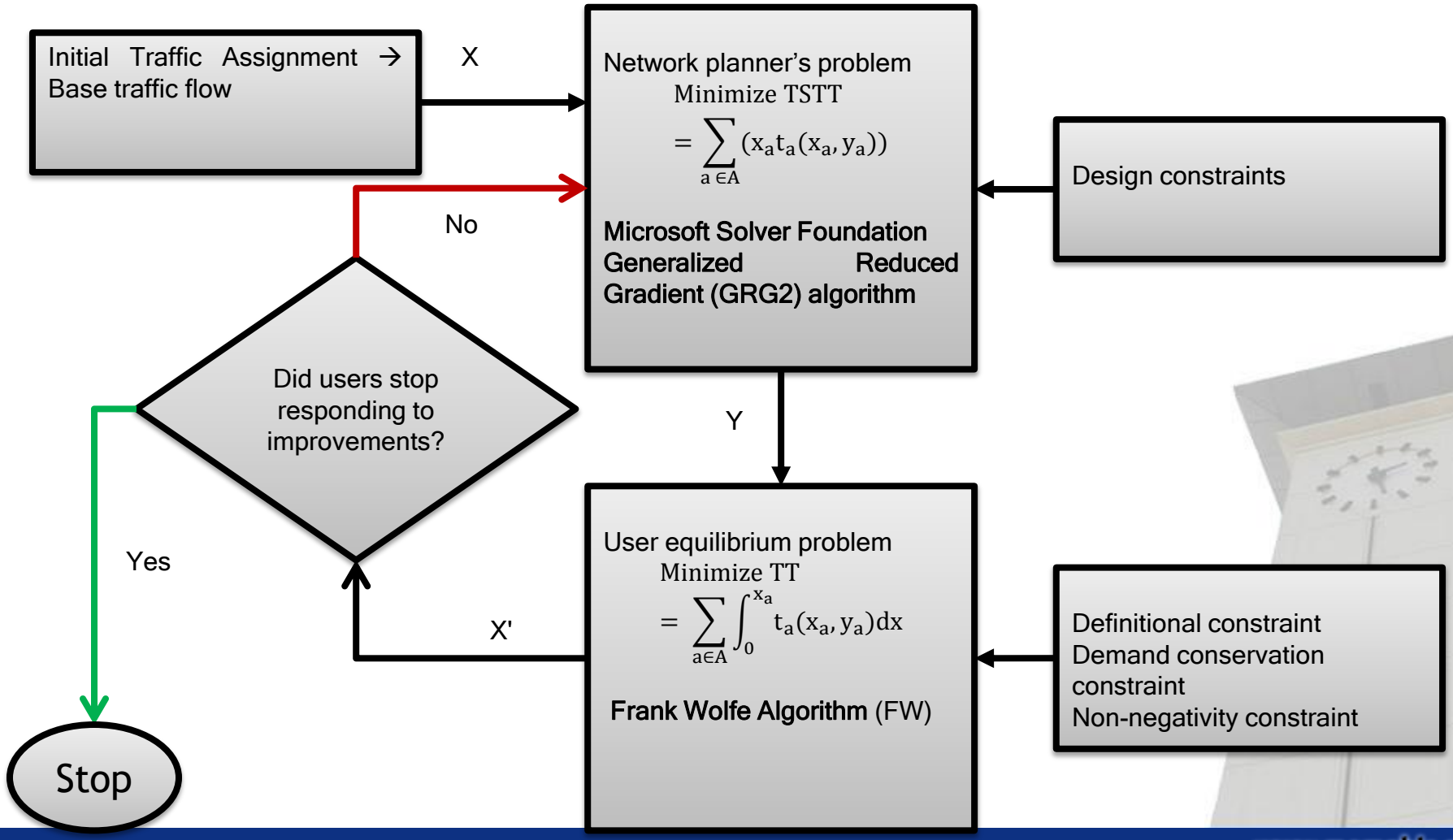
$$f_k^{ij} \geq 0, \;\; \forall_k \in k^{ij}, \qquad \forall (i,j) \in IJ,$$

$$q_{ij} \geq 0, \quad \forall (i,j) \in IJ$$

Notations

C_a	:	The capacity for link a
f_{ij}^r	:	Flow on path *r*, connecting each Origin-Destination (O-D) pair (*i*-*j*)
q_{ij}	:	Demand between each Origin-Destination (O-D) pair (*i*-*j*)
t_a	:	Travel time for link a
$t_a(x_a, y_a)$	:	Travel cost on link a as a function of flow and capacity expansion
x_a	:	Flow for link a
α_a	:	Constant, varying by facility type (BPR function)
β_a	:	Constant, varying by facility type (BPR function)
$\delta_{a,ij}^r$	:	binary variable 0,1 {1,if link a ∈A is on path k ∈ k^ij:0,otherwise}
t_o	:	Free flow time on link a
y_a	:	Capacity expansion for link ‘a’ (nonnegative real value)

Flow Chart


Initial Traffic Assignment → Base traffic flow
X
Network planner's problem
Minimize TSTT
$= \sum_{a \in A} (x_a t_a(x_a, y_a))$
Microsoft Solver Foundation Generalized Reduced Gradient (GRG2) algorithm
Design constraints
No
Did users stop responding to improvements?
Y
Yes
X'
User equilibrium problem
Minimize TT
$= \sum_{a \in A} \int_0^{x_a} t_a(x_a, y_a) dx$
Frank Wolfe Algorithm (FW)
Definitional constraint
Demand conservation constraint
Non-negativity constraint
Stop

LLP

- **Frank Wolfe Algorithm (FW)**
- SOURCE: YOSEF SHEFFI, Massachusetts Institute of Technology

Step 0: Initialization

- Perform all-or-nothing assignment based on ta =ta (0) ∀a. A new flow vector {xa} will be generated. Set counter n = 1.

Step 1: Update

- Set ta=ta (xa) ∀a

Step 2: Finding Direction

- Perform all-or-nothing assignment based on {ta}.
- A new auxiliary flow vector {x'a} will be generated.

Step 3: Line search

- Find αn (0 ≤α ≤1) that solves equation :
- $\min z(x) = \sum_a \int_0^{x_a + \propto (x'^n_a - x^n_a)} t_a(w)dw$

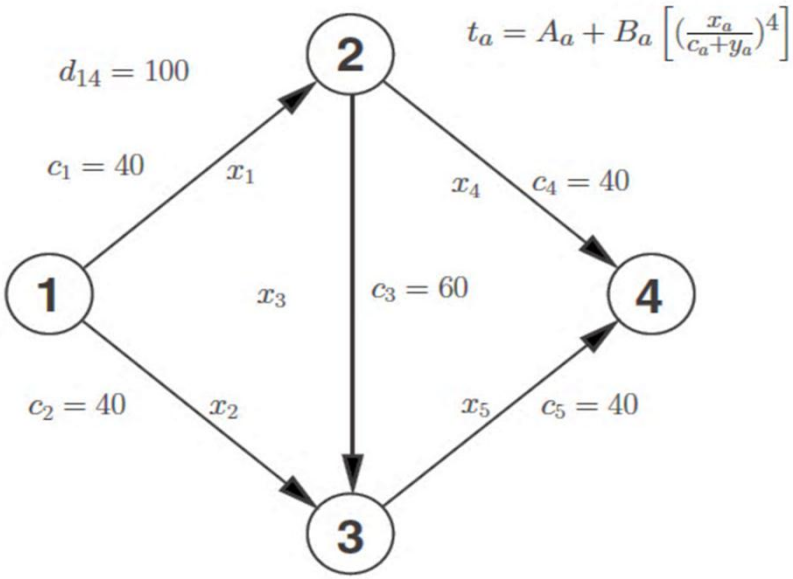
Step 4: Move

- $x_a^{n+1} = x_a^n + \propto_n (x'^n_a - x_a^n), \forall a$

Step 5: Convergence test

- $\frac{\sqrt{\sum_a (x_a^{n+1} - x_a^n)^2}}{\sum_a x_a^n} \leq k$

Test Network 1



$$t_a(x_a, y_a) = A_a + B_a\left(\frac{x_a}{C_a + y_a}\right)^4$$

$$TSTT(y) = \sum_a (t_a(x_a, y_a).x_a + 1.5 d_a y_a^2$$

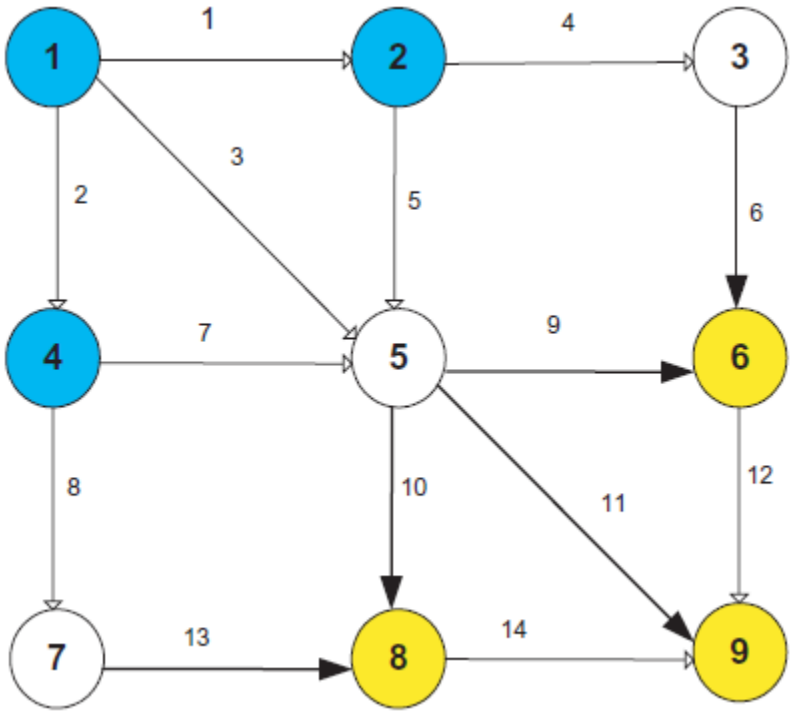
Arc a	A_a	B_a	C_a	d_a
1	4	0.60	40	2
2	6	0.90	40	2
3	2	0.30	60	1
4	5	0.75	40	2
5	3	0.45	40	2

Comparison of Results

	Case	MINOS	Hooke-Jeeves (H-J)	EDO	GA	Current Study
1	Demand =100					
	y_1	1.34	1.25	1.31	1.33	**1.33**
	y_2	1.21	1.20	1.19	1.22	**1.21**
	y_3	0.00	0.00	0.06	0.02	**0.00**
	y_4	0.97	0.95	0.94	0.96	**0.96**
	y_5	1.10	1.10	1.06	1.10	**1.10**
	Z	1200.58	1200.61	1200.64	1200.58	**1200.58**
2	Demand =150					
	y_1	6.05	5.95	5.98	6.08	**6.06**
	y_2	5.47	5.64	5.52	5.51	**5.46**
	y_3	0.00	0.00	0.02	0.00	**0.00**
	y_4	4.64	4.60	4.61	4.65	**4.64**
	y_5	5.27	5.20	5.27	5.27	**5.27**
	Z	3156.21	3156.38	3156.24	3156.23	**3156.21**
3	Demand =200					
	y_1	12.98	13.00	12.86	13.04	**12.98**
	y_2	11.73	11.75	12.02	11.73	**11.73**
	y_3	0.00	0.00	0.02	0.01	**0.00**
	y_4	10.34	10.25	10.33	10.33	**10.34**
	y_5	11.74	11.75	11.77	11.78	**11.74**
	Z	7086.12	7086.21	7086.45	7086.16	**7086.11**
4	Demand =300					
	y_1	28.45	28.44	28.11	28.48	**28.47**
	y_2	25.73	25.75	26.03	25.82	**25.71**
	y_3	0.00	0.00	0.01	0.08	**0.00**
	y_4	23.40	23.44	23.39	23.39	**23.41**
	y_5	26.57	26.56	26.58	26.48	**26.55**
	Z	21209.90	21209.91	21210.54	21210.06	**21209.90**

	Names of heuristics	Sources
GA	Genetic Algorithm	Mathew (2009)
H-J	Hooke-Jeeves algorithm	Abdulaal and LeBlanc (1979)
EDO	Equilibrium Decomposed Optimization (Bolzano search)	Suwansirikul et al. (1987)
MINOS	Modular In-core Non linear System	Suwansirikul et al. (1987)

Test Network 2



Source: Chiou et al (2005)

O/D	6	8	9
1	120	150	100
2	130	200	90
4	80	180	110

$$t_a(x_a, y_a) = A_a + B_a\left(\frac{x_a}{C_a + y_a}\right)^4$$

$$TSTT(y) = \sum_a (t_a(x_a, y_a).x_a + \theta y_a$$

Link a	Aa	Ba	Ca	θa
1	2	0.3	280	–
2	1.5	0.225	290	–
3	3	0.45	280	–
4	1	0.15	280	–
5	1	0.15	600	–
6	2	0.3	300	1.5
7	2	0.3	500	–
8	1	0.15	400	–
9	1.5	0.225	500	1
10	1	0.15	700	1
11	2	0.3	250	1.5
12	1	0.15	300	–
13	1	0.15	350	1
14	1	0.15	220	–

Comparison of Results

Case	SAB	GP	CG	QNEW	PT	Current Study
y6	0	0	0	0	0	0
y9	0	0	0	0	0	0
y10	0	0	0	0	0	0
y11	0	0	0	0	0	0
y13	128.087	128.103	137.445	137.445	127.793	75.149
Zy	**4110.02**	**4110.02**	**4109.55**	**4109.55**	**4110.05**	**3891.685**

GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method
SAB	Sensitivity Analysis Based

Source: Chiou et al (2005)

Comparison of Results

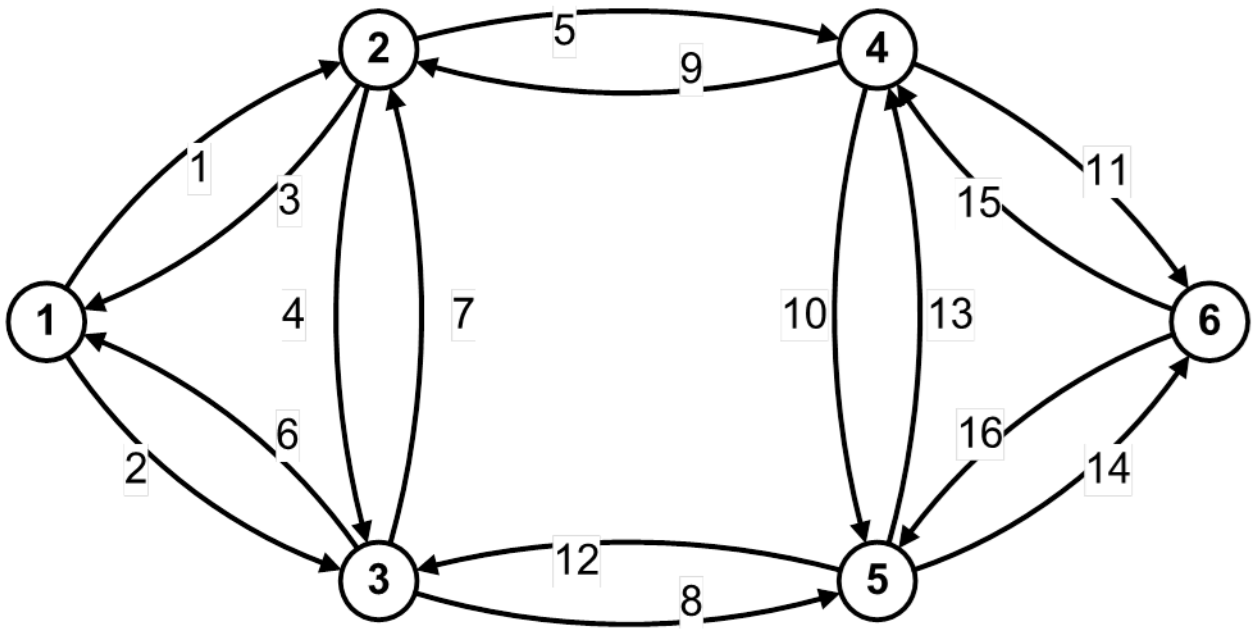
- Comparison of results on 9-node grid network with scaling factors

Scalar	SAB	GP	CG	QNEW	PT	Current Study
0.8	3043.18	3042.91	3042.91	3042.91	3043.27	3027.90
0.9	3549.36	3549.02	3549.02	3549.02	3549.45	3451.21
1.1	4634.97	4632.38	4632.38	4632.38	4632.38	4346.82
1.2	5137.64	5135.28	5135.29	5135.29	5135.29	4729.88
1.3	5700.01	5703.21	5697.22	5697.22	5697.22	5378.90
1.4	6240.73	6244.02	6237.98	6237.98	6237.98	5974.95
1.5	6858.2	6781.51	6781.51	6781.51	6781.51	6544.21

GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method
SAB	Sensitivity Analysis Based

Test Network 3

O/D	6	1
1	5	-
6	-	10



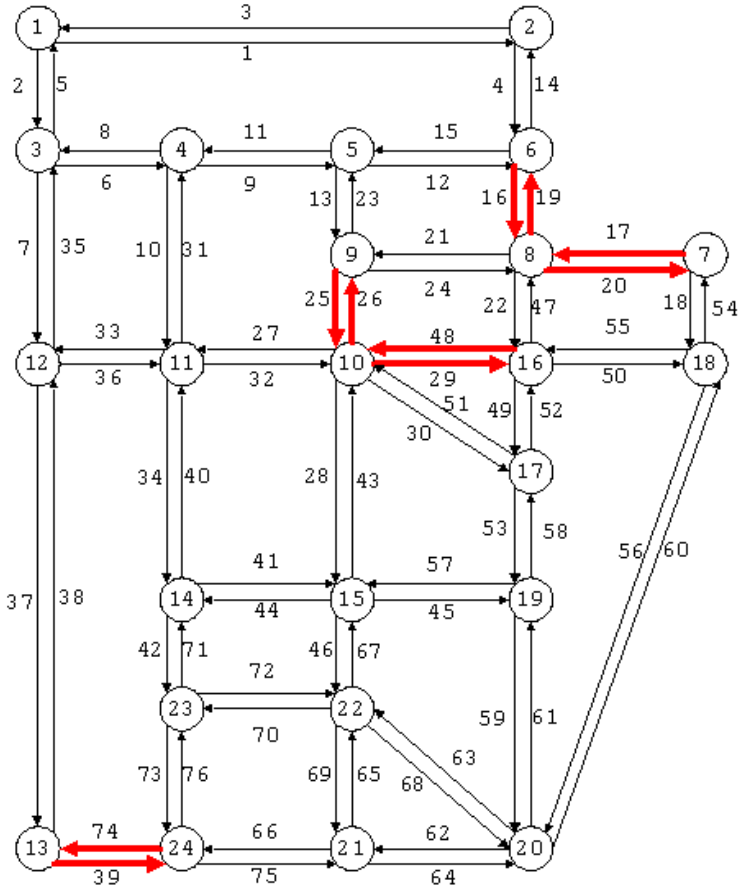
Comparison of Results

Case 1					
Demand (1,6)=5.0					
Demand (6,1)=10.0					
	MINOS	H-J	EDO	IOA	Current Study
y_1					
y_2					
y_3		1.2	0.13		
y_4					
y_5					
y_6	6.58	3	6.26	6.95	4.6
y_7					
y_8					
y_9					
y_{10}					
y_{11}					
y_{12}					
y_{13}					
y_{14}					
y_{15}	7.01	3	0.13	5.66	8.23
y_{16}	0.22	2.8	6.26	1.79	0
Z	211.25	215.08	201.84	210.86	213.24

	Names of heuristics	Sources
IOA	Iterative Optimization-Assignment algorithm	Allsop (1974)
H-J	Hooke-Jeeves algorithm	Abdulaal and LeBlanc (1979)
EDO	Equilibrium Decomposed Optimization (Bolzano search)	Suwansirikul et al. (1987)
MINOS	Modular In-core Non linear System	Suwansirikul et al. (1987)

Test Network 4

candidate links are marked by red arrow



Sioux Falls Network

Comparison of Results

Comparison of Results for Sioux Falls Network

Case	H-J	H-J	EDO	SA	SAB	GP	CG	QNew	PT	GA	Current Study
y16	4.8	3.8	4.59	5.38	5.74	4.87	4.77	5.3	5.02	5.17	**5.13**
y17	1.2	3.6	1.52	2.26	5.72	4.89	4.86	5.05	5.22	2.94	**1.35**
y19	4.8	3.8	5.45	5.5	4.96	1.87	3.07	2.44	1.83	4.72	**5.13**
y20	0.8	2.4	2.33	2.01	4.96	1.53	2.68	2.54	1.57	1.76	**1.32**
y25	2	2.8	1.27	2.64	5.51	2.72	2.84	3.93	2.79	2.39	**2.98**
y26	2.6	1.4	2.33	2.47	5.52	2.71	2.98	4.09	2.66	2.91	**2.98**
y29	4.8	3.2	0.41	4.54	5.8	6.25	5.68	4.35	6.19	2.92	**4.89**
y39	4.4	4	4.59	4.45	5.59	5.03	4.27	5.24	4.96	5.99	**4.45**
y48	4.8	4	2.71	4.21	5.84	3.76	4.4	4.77	4.07	3.63	**4.97**
y74	4.4	4	2.71	4.67	5.87	3.57	5.52	4.02	3.92	4.43	**4.4**
Zy	82.5	82.61	84.5	81.89	84.38	84.15	84.86	83.19	84.19	81.74	**80.99**

GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method

Comparison of Results

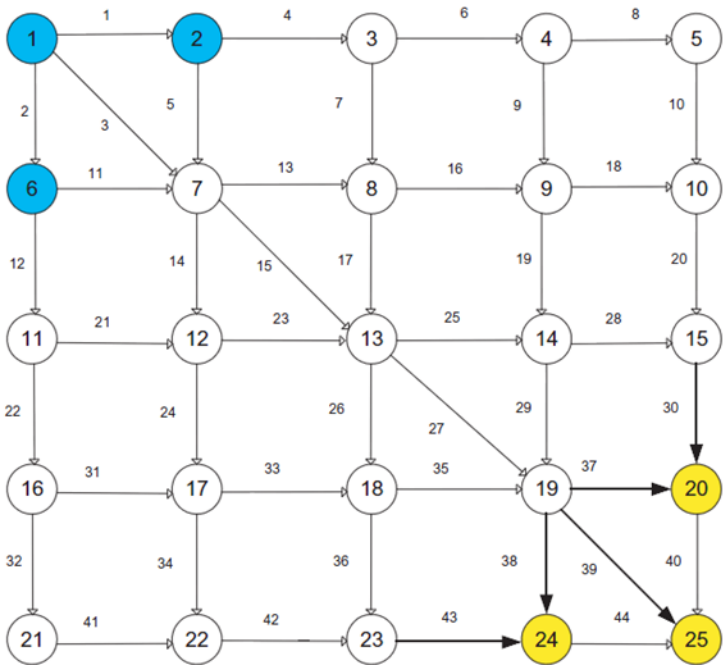
Comparison of Results for Sioux Falls Network for different demand level

Scalar	SAB	GP	CG	QNew	PT	EDO	IOA	GA	Current Study
0.8	51.76	48.38	48.78	48.84	48.81	49.51	53.58	48.92	**48.15**
FW Itr.	14	10	3	4	9	7	28	59	**29**
1	84.21	82.71	82.53	83.07	82.53	83.57	87.34	81.74	**80.99**
FW Itr.	11	9	6	4	7	12	31	77	**35**
1.2	144.86	141.53	141.04	141.62	142.27	149.39	150.99	137.92	**135.80**
FW Itr.	9	11	10	7	9	12	31	67	**36**
1.4	247.8	246.04	246.04	242.74	241.08	253.39	279.39	232.76	**229.22**
FW Itr.	15	9	6	5	7	17	16	78	**36**
1.6	452.01	433.64	408.45	409.04	431.11	427.56	475.08	390.54	**380.91**
FW Itr.	14	9	9	9	11	19	40	83	**40**

GA	Genetic Algorithm
GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method

Test Network 5

O/D	20	24	25
1	120	150	100
2	130	200	90
6	80	180	110



$$t_a(x_a, y_a) = A_a + B_a\left(\frac{x_a}{C_a + y_a}\right)^4$$

$$TSTT(y) = \sum_a (t_a(x_a, y_a).x_a + \theta\, y_a$$

Linka	Aa	Ba	Ka	θa	Linka	Aa	Ba	Ka	θa
1	2	0.3	280	–	23	2	0.3	500	–
2	1.5	0.225	290	–	24	1	0.15	700	–
3	3	0.45	280	–	25	1.5	0.225	500	–
4	2	0.3	280	–	26	1	0.15	700	–
5	1	0.15	600	–	27	2	0.3	250	–
6	1	0.15	280	–	28	1.5	0.225	500	–
7	1	0.15	600	–	29	1	0.15	700	–
8	1	0.15	280	–	30	1	0.15	300	1
9	1	0.15	600	–	31	2	0.3	500	–
10	2	0.3	300	–	32	1	0.15	400	–
11	2	0.3	500	–	33	2	0.3	500	–
12	1.5	0.225	290	–	34	1	0.15	700	–
13	2	0.3	500	–	35	1.5	0.225	500	–
14	1	0.15	600	–	36	1	0.15	700	–
15	3	0.45	280	–	37	1.5	0.225	500	1
16	1.5	0.225	500	–	38	1	0.15	700	1
17	1	0.15	600	–	39	2	0.3	250	1.5
18	1.5	0.225	500	–	40	1	0.15	300	–
19	1	0.15	600	–	41	1	0.15	350	–
20	2	0.3	300	–	42	1	0.15	350	–
21	2	0.3	500	–	43	1	0.15	220	1
22	1	0.15	400	–	44	1	0.15	220	–

Comparison of Results

- Comparison of results on 25-node grid network with scaling factors

Scalar	SAB	GP	CG	QNEW	PT	Current Study
0.8	7590.19	7590.06	7590.19	7590.19	7590.36	8108.75
0.9	8888.89	8888.77	8888.91	8888.91	8889.07	9226.66
1.1	11452	11440.9	11441.1	11441.1	11441.3	11378.61
1.2	12643.1	12632.5	12626.3	12626.3	12626.3	12500.24
1.3	13833.7	13831.6	13831.7	13831.7	13832.1	13683.70
1.4	15195.7	15185.6	15185.7	15185.7	15186.4	14937.38
1.5	16371.6	16354.6	16354.6	16354.6	16354.8	16253.29

GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method

Questions